A High Precision Test of the Equivalence Principle

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What and where is CENPA?
A close look at the N part of campus

Now: CENPA Center for Experimental Nuclear Physics and Astrophysics

Gravity lab

Machine shop
Inside CENPA

Check it out:  www.npl.washington.edu
Current Experiments and Research

- Neutrino Physics
  - **KATRIN** Direct neutrino mass measurement
  - **Majorana** Double beta decay experiment
  - **Sudbury Neutrino Observatory** Solar neutrino measurement
  - **SNO+** Double beta decay experiment

- Determination of the $^{22}$Na($p$,γ) cross section

- Fundamental Symmetries in Nuclear Physics
  - **Scalar interactions: $^{32}$Ar**
  - **Double beta decay matrix elements: $^{100}$Tc**
  - **Double beta decay matrix elements: $^{116}$Cd**
  - **Ultra Cold Neutrons - at LANSCE**

- **Eötvös Group** Sub-Newtonian Gravitational Forces Experiments

- Axions
  - **ADMX** Axion dark matter experiments
  - **Torsion pendulum experiment**

- **UW NPL Ultra-Relativistic Heavy Ion Experiments**

- $^{199}$Hg Electric Dipole Moment

- **Radiation interactions with matter**

http://www.npl.washington.edu/research.html
Eötvash group

Eric Adelberger
Jens Gundlach
Blayne Heckel
Stephan Schlamminger
Erik Swanson
Frank Fleischer
Seth Hoedl
Ted Cook
Charles Hagedorn
William Terrano
Matt Turner
Todd Wagner

- Do all bodies drop with the same acceleration?
- Is there a preferred frame or direction in the universe?
- Is there a large (>μm) extra dimension?

(02/18/10 Talk by Charles Hagedorn)

- Search for Axions.
- Crucial measurements for LISA (Laser Interferometer Space Antenna).
- Research for LIGO

Torsion balance

Graduate students
Mass does not add up
Mass does not add up

Proton + Electron = $^1\text{H}$

938 272 013 eV/c$^2$ + 510 999 eV/c$^2$ ≤ 938 782 999 eV/c$^2$
Mass does not add up

Proton + Electron = \(^1\text{H}\)

938 272 013 eV/c\(^2\) + 510 999 eV/c\(^2\) \(\leq\) 938 782 999 eV/c\(^2\)

938 272 013 eV/c\(^2\) + 510 999 eV/c\(^2\) \(\equiv\) 938 783 012 eV/c\(^2\)

Mass defect = binding Energy

Binding Energy = \(\Delta E_{\text{pot}} - \Delta E_{\text{kin}}\)

13.6 eV/c\(^2\)
The non-linearity is rather small

but occurs also in gravitationally bound systems

Mass defect for 1 kg of Earth: $\Delta m = 0.46 \mu g$

Mass defect for 1 kg of Moon: $\Delta m = 0.02 \mu g$
The mass of an object

\[ m = \sum m_c + \sum \frac{E_{\text{kin}}}{c^2} - \sum \frac{E_{\text{pot}}}{c^2} \]
Newton’s Principia (1689)

Newton’s 2\textsuperscript{nd} Law

\[ F_i = m_i \ a \]

Gravitational Law

\[ F_G = G \ m_{g1} \ m_{g2} / r^2 \]
Newton’s Principia (1689)

Newton’s 2nd Law

\[ F_i = m_i a \]

Gravitational Law

\[ F_G = G \frac{m_{g1} m_{g2}}{r^2} \]

Equivalence Principle (EP):

\[ m_i = m_g \]
Newton’s Principia (1689)

Newton’s 2nd Law

\[ F_i = m_i a \]

Gravitational Law

\[ F_G = G \frac{m_{g1} m_{g2}}{r^2} \]

Equivalence Principle (EP): \[ m_i = m_g \]

Weak Equivalence Principle:
Gravitational binding energy is excluded.

Strong Equivalence Principle:
Includes all 4 fundamental interactions.
The Equivalence Principle

Our Current theory of gravity, General Relativity, is based on the **Equivalence Principle**: All bodies fall in a gravitational field with the same acceleration regardless of their mass or internal structure.

Einstein realized, that:

It is impossible to distinguish between an uniform gravitational field and an acceleration.
The Equivalence Principle

Gravitational field $g$  
\[ \vec{F} = mg \]

Acceleration $a$  
\[ \vec{F} = -ma \]

Inertial mass = gravitational mass, $m_I = m_G$ for all bodies
General Relativity and the EP

Three classical tests:

- Perihelion shift of Mercury
- Deflection of light by the Sun
- Gravitational red shift of light

The last two can be explained and calculated with the EP alone!
The known unknowns

- GR is not a quantum theory.
- Cosmological constant problem.
- Dark matter.

The unknown unknowns

- Is there another force (5\textsuperscript{th} force)?
- ...

EP-Tests provide a big bang for the buck!
Searching for New Interactions

Good model for a new interaction:

\[ V(r) = \alpha G \left( \frac{q_1}{\mu_1} \right) \left( \frac{q_2}{\mu_2} \right) \frac{m_1 m_2}{r} e^{-r/\lambda} \]

Strength relative to gravity

Interaction range

Source

Test mass

Assumed charge

<table>
<thead>
<tr>
<th>Beryllium</th>
<th>Titanium</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>q/\mu</td>
<td>q/\mu</td>
<td></td>
</tr>
<tr>
<td>Baryon number B</td>
<td>0.99868</td>
<td>1.001077</td>
</tr>
<tr>
<td></td>
<td>Mass (u)</td>
<td>q=B</td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td>-----</td>
</tr>
<tr>
<td>$^1_1p$</td>
<td>1.0073</td>
<td>1</td>
</tr>
<tr>
<td>$^1_0n$</td>
<td>1.0087</td>
<td>1</td>
</tr>
<tr>
<td>$^9_4Be$</td>
<td>9.1012</td>
<td>9</td>
</tr>
<tr>
<td>$^{48}_{22}Ti$</td>
<td>47.9479</td>
<td>48</td>
</tr>
</tbody>
</table>
$B/\mu$ varies, because
1st Tests of the Equivalence Principle

\[ F = m_G g \]
\[ F = m_I a \]
\[ a = \frac{m_G}{m_I} g \]

Time \( t \) to fall from \( h \):

\[ t = \sqrt{\frac{2h}{m_G / m_I}} g \]

1600 Galileo:

\[ \eta = \frac{a_1 - a_2}{\frac{1}{2} (a_1 + a_2)} \approx 0.1 \]
2nd Generation Tests

Measurement of the swing periods of pendula:

\[ T = 2\pi \sqrt{\frac{L}{g} \frac{m_I}{m_G}} \]

Newton (1686), Bessel (1830), Potter (1923)

\[ \eta \approx 2 \times 10^{-5} \]
$F_I = m_I \omega^2 r \cos \theta$

$F_G = m_G g$

$\varepsilon = \frac{m_I \omega^2 r}{m_G g} \sin(2\theta)$

$G \approx \frac{m_I \omega^2 r}{m_G 2g}$

Eötvös Experiments
The torsion balance

• A violation of the EP would yield to different plumb-line for different materials.

• A torsion balance can be used to measure the difference in plumb-lines:

  Torsion fiber hangs like the average plumb line.

  Difference in plumb lines produces a torque on the beam.

  -> twist in the fiber

  Eötvös (1922)  \[ \eta \approx 5 \times 10^{-9} \]
Dicke’s idea

Using the Sun as a source

\[ \eta \approx 1 \times 10^{-11} \]
Historical overview

\[ \eta = \frac{|a_1 - a_2|}{\frac{1}{2}(a_1 + a_2)} \]

Galileo
1600

Bessel
1700

Eötvös
1800

Potter
1900

Dicke
2000

UW

Type of experiment
drop
pendula
torsion balance
modulated torsion balance
Principle of our experiment

Composition dipole pendulum
(Be-Ti)

Rotation
1 rev./ 20min

Autocollimator (=optical readout)

<table>
<thead>
<tr>
<th>source mass</th>
<th>λ (m)</th>
</tr>
</thead>
</table>
| local masses (hill)       | 1 - 10^4| modulated
| entire earth              | 10^6 - 10^7|
| Sun                       | 10^{11} - ∞|
| Milky Way (incl. DM)      | 10^{20} - ∞|
EP torsion pendulum

20 μm diameter tungsten fiber
(length: 108 cm)
$K=2.36 \text{nNm}$

8 test masses (4 Be & 4 Ti)
4.84 g each (within 0.1 mg)
(can be removed)

4 mirrors

tuning screws for adjusting
tiny asymmetries

torsional frequency: 1.261 mHz
quality factor: 4000
decay time: 11d 6.5 hrs
machining tolerance: 5 μm
total mass: 70 g
The upper part of the apparatus

- feedthrough for electric signals
- thermal insulation
- air bearing turntable
- electronics of angle encoder
- thermal expansion feet to level turntable
The lower part of the apparatus

- Vacuum chamber \((10^{-5} \text{ Pa})\)
- Ion pump
- Support structure for gravity gradient compensators
- Autocollimator
- Gravity gradient compensators
Raw-data

Signal + free Oscillation

ω_{SIG} = \frac{2}{3} \omega_{PEND} = \frac{2\pi}{(1200 \text{ s})}

θ Pendulum's Excursion (μrad)

ϕ Turntable angle (deg)
Filter: \( F(t) = \theta(t-T/4) + \theta(t+T/4) \)

These data were taken before the pendulum was trimmed. The signal is a result of the gravitational coupling.
Data reduction
A complete day of data

\[ \sigma \approx 3 \text{ nrad} \sqrt{\text{day}} \]
Differential acceleration

Deflection from equilibrium: \( \phi \)

is caused by a torque: \( \tau = \kappa \phi \)

\[ \tau = F_1 d - F_2 d \]

\[ \tau = ma_1 d - ma_2 d = md(a_1 - a_2) = md\Delta a \]

\[ \Delta a = \frac{\kappa}{md} \phi \]
Some numbers

\[ \Delta a = \frac{\kappa}{4md} \varphi \]

\[ \kappa = 2.36 \times 10^{-9} \text{ Nm} \]
\[ m = 4.84 \times 10^{-3} \text{ kg} \]
\[ d = 1.9 \times 10^{-2} \text{ m} \]

6.41 x 10^{-15} m/s^2 / nrad

Can be measured with
An uncertainty of 3 nrad per day

\[ \Delta a \] can be measured to
20 fm/s^2
A feel for numbers

φ = 3 nrad

Δx = ?

Δa = 20 fm/s²

Δh = ?

g = 9.81 m/s²

g = 9.81 m/s² - Δa
A feel for numbers

φ = 3 nrad

Δx = 3 mm

Δa = 20 fm/s²

Δh = 7 nm

g = 9.81 m/s²  g = 9.81 m/s² - Δa
We rotate the pendulum (and thus the dipole) in respect to the readout every day by 180°.
The 12 days of data from the previous slide are condensed into a single point.

Dipole is **orthogonal** to readout.

Dipole is (anti-) **parallel** to readout.

Resolved signal in WE-direction!

Rotating every day,

<table>
<thead>
<tr>
<th>Run</th>
<th>$\Delta a_{WE}$ (fm/s²)</th>
<th>$\Delta a_{NS}$ (fm/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-10 ± 2</td>
<td>-10 ± 2</td>
</tr>
<tr>
<td>2</td>
<td>-10 ± 2</td>
<td>-10 ± 2</td>
</tr>
<tr>
<td>3</td>
<td>-10 ± 2</td>
<td>-10 ± 2</td>
</tr>
<tr>
<td>4</td>
<td>-10 ± 2</td>
<td>-10 ± 2</td>
</tr>
<tr>
<td>5</td>
<td>-10 ± 2</td>
<td>-10 ± 2</td>
</tr>
</tbody>
</table>
After 2 months of data taking and systematic checks we physically invert the dipole on the pendulum and put it back into the apparatus.

These data points have been corrected for systematic effects.

Only statistical uncertainties shown.
## Corrected result

<table>
<thead>
<tr>
<th></th>
<th>$\Delta a_{N,Be-TI}$ \hspace{2cm} \left(10^{-15} \text{ m/s}^2\right)$</th>
<th>$\Delta a_{W,Be-TI}$ \hspace{2cm} \left(10^{-15} \text{ m/s}^2\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>as measured</td>
<td>$3.3 \pm 2.5$</td>
<td>$-2.4 \pm 2.4$</td>
</tr>
<tr>
<td>Due to gravity gradients</td>
<td>$1.6 \pm 0.2$</td>
<td>$0.3 \pm 1.7$</td>
</tr>
<tr>
<td>Tilt induced</td>
<td>$1.2 \pm 0.6$</td>
<td>$-0.2 \pm 0.7$</td>
</tr>
<tr>
<td>Temperature gradients</td>
<td>$0 \pm 1.7$</td>
<td>$0 \pm 1.7$</td>
</tr>
<tr>
<td>Magnetic coupling</td>
<td>$0 \pm 0.3$</td>
<td>$0 \pm 0.3$</td>
</tr>
<tr>
<td>Corrected</td>
<td>$0.6 \pm 3.1$</td>
<td>$-2.5 \pm 3.5$</td>
</tr>
</tbody>
</table>
Gravity gradients (1/4)

Gravitational potential energy between the pendulum and the source masses is given by

\[ w = -4 \pi G \sum_{l=0}^{\infty} \frac{1}{2l + 1} \sum_{m=-l}^{l} Q_{lm} q_{lm} e^{-im\phi} \].

\[ Q_{lm} = \int d^3r' \rho_{\text{source}}(\mathbf{r}') r'^{-(l+1)} Y_{lm}(\mathbf{r}') \quad \text{gravity gradient field} \]

\[ q_{lm} = \int d^3r \rho_{\text{pend}}(\mathbf{r}) r^{l} Y_{lm}^*(\mathbf{r}) \quad \text{gravity multipole moment} \]

Torque for $1\omega$ ($m=1$) signal:

\[ \tau = 8 \pi G \frac{1}{2l + 1} |q_{l1}| |Q_{l1}| \]

\[ q_{11} \]

\[ q_{21} \]

\[ q_{31} \]

not possible for a torsion pendulum
Gravity gradients (2/4)

$Q_{21}$ compensators
Total mass: 880 kg
$Q_{21} = 1.8 \text{ g/cm}^3$

$Q_{31}$ compensators
Total mass: 2.4 kg
$Q_{31} = 6.7 \times 10^{-4} \text{ g/cm}^4$

360° rotatable

hillside & local masses
Gravity gradients (3/4)

$q_{41}$ configuration

$q_{21}$ configuration installed
Gravity gradients (4/4)

Without compensation:

\[ Q_{21} = 1.78 \text{ gcm}^{-3} \]

Uncertainty due to fluctuating fields:

\[ 0.2 \text{ fm/s}^2 \text{ N} \]
\[ 1.7 \text{ fm/s}^2 \text{ W} \]
Tilt coupling
Tilt of the suspension point + anisotropies of the fiber will rotate.
Tilt coupling

Tilt of the suspension point + anisotropies of the fiber will rotate

1. Thicker fiber at the top provides more torsional stiffness
Tilt coupling

1. Thicker fiber at the top provides more torsional stiffness
2. Feedback minimizes tilt of TT

Tilt of the suspension point + anisotropies of the fiber will rotate
More tilt

Active foot

PID

No signal!

1.70m

0.23m

53 nrad

Remaining tilt uncertainty:

0.6 fm/s^2 N
0.7 fm/s^2 W
Thermal

Effect on the applied gradient on the signal (measurement was done on one mirror):

50 nrad Signal

\[ \Delta T = 7 \text{K} \]

During data taking
\[ \Delta T \approx 0.053 \text{K} \]

\[ \Rightarrow 0.38 \text{ nrad} \]

1.7 fm/s^2 N

1.7 fm/s^2 W

Effect of a 7 K gradient on two different mirrors.
Recent improvements

Lamp on a ruler used to find the sensitive spot.

Highest sensitivity to the lamp at 41 in.

Additional insulation (@ 41 in) reduced the temperature feedthrough.
Interpreting our result

North: \( a_{\text{Be}} - a_{\text{Ti}} = (0.6 \pm 3.1) \times 10^{-15} \text{ m/s}^2 \)

West: \( a_{\text{Be}} - a_{\text{Ti}} = (-2.5 \pm 3.5) \times 10^{-15} \text{ m/s}^2 \)
Interpreting our result

North: \( a_{Be} - a_{Ti} = (0.6 \pm 3.1) \times 10^{-15} \text{ m/s}^2 \)

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\[ \eta = \frac{|a_1 - a_2|}{\frac{1}{2} |a_1 + a_2|} \]
Interpreting our result

North: \( a_{\text{Be}} - a_{\text{Ti}} = (0.6 \pm 3.1) \times 10^{-15} \text{ m/s}^2 \)

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\[ \eta = \frac{|a_1 - a_2|}{\frac{1}{2} |a_1 + a_2|} \]

\( \eta(\text{Be-Ti}) = (0.3 \pm 1.8) \times 10^{-13} \)
Results

Charge: B

Strength relative to gravity

Range of a hypothetical “fifth” force (m)

10^{-4}

10^{-5}

10^{-6}

10^{-7}

10^{-8}

10^{-9}

10^{-10}

10^{-11}

10^{0}

10^{3}

10^{6}

10^{9}

10^{12}

excluded with 95% confidence

EW99

EW94

PU64

MSU72

EW08-Combined

EW99

EW94

LLR04

EW08-Combined

q = B

17 x
Acceleration to the center of our galaxy

1825 h of data taken over 220 days

Differential acceleration to the center of the galaxy:
\((-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2\)

In quadrature:
\((2.7 \pm 3.1) \times 10^{-15} \text{ m/s}^2\)

Hypothetical signal:
(7 times larger)
\(20 \times 10^{-15} \text{ m/s}^2\)
Galactic dark matter

Earth

Milky Way
Galactic dark matter

Earth

Milky Way

Halo of dark matter
Galactic dark matter

Dark matter inside the Earth’s galactic orbit causes 25-30% of the total acceleration.

Halo of dark matter
Galactic dark matter

Our acceleration towards the galactic center is:

\[ a_{\text{gal}} = a_{\text{dark}} + a_{\text{ordinary}} = 1.9 \times 10^{-10} \text{ m/s}^2 \Rightarrow a_{\text{dark}} = 5 \times 10^{-11} \text{ m/s}^2 \]

Dark matter inside the Earth’s galactic orbit causes 25-30% of the total acceleration.

Our acceleration towards the galactic center is:

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Dark matter inside the Earth’s galactic orbit causes 25-30% of the total acceleration.

Measured differential acceleration towards the galactic center is:

\[ \Delta a_{\text{gal}} = (-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2 \Rightarrow \eta_{\text{dark}} = \left| \Delta a_{\text{gal}} / a_{\text{dark}} \right| = (-4 \pm 7) \times 10^{-5} \]
Our acceleration towards the galactic center is:

\[ a_{\text{gal}} = a_{\text{dark}} + a_{\text{ordinary}} = 1.9 \times 10^{-10} \text{ m/s}^2 \implies a_{\text{dark}} = 5 \times 10^{-11} \text{ m/s}^2 \]

Measured differential acceleration towards the galactic center is:

\[ \Delta a_{\text{gal}} = (-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2 \implies \eta_{\text{dark}} = \frac{|\Delta a_{\text{gal}}|}{a_{\text{dark}}} = (-4 \pm 7) \times 10^{-5} \]

The acceleration of Be and Ti towards dark matter does not differ by more than 150 ppm (with 95% confidence).
Summary

- Test of the equivalence principle with a rotating torsion balance.
- Principle of the measurement
- Main systematic effects

Results
- Earth fixed (North): $a_{\text{Be}} - a_{\text{Ti}} = (0.6 \pm 3.1) \times 10^{-15} \text{ m/s}^2$.
- $\eta = (0.3 \pm 1.8) \times 10^{-13}$.
- Towards Galaxy: $a_{\text{Be}} - a_{\text{Ti}} = (-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2$.
- $\eta_{\text{DM}} = (-4 \pm 7) \times 10^{-5}$.
- 10x improved limits on a long range interaction.
Outlook

- Ideas for improvement:
  - Decouple damper systematic from pendulum.
  - Integrated $q_{21}$ measurement.
  - Higher Q fiber to reduce thermal noise

- Earth-like and moon-like TB to keep pace with improvements in LLR.
Acknowledgement

Jens Gundlach
Todd Wagner
Ki-Young Choi
Eric Adelberger
This talk stops here.
Searching for New Interactions

Yukawa potential

\[ V(r) = \frac{g^2}{4\pi} \frac{Q_1 Q_2}{r_{12}} \frac{1}{r_{12}} e^{-r_{12}/\lambda} \]

\[ Q_x = Nq_x = \frac{m_x}{u\mu_x} q_x \]

1 u = 1.66 x 10^{-27} kg

\[ V(r) = \frac{g^2}{u^2 4\pi} \left( \frac{q_1}{\mu_1} \right) \left( \frac{q_2}{\mu_2} \right) \frac{1}{r_{12}} e^{-r_{12}/\lambda} \]

\[ V(r) = \alpha G \left( \frac{q_1}{\mu_1} \right) \left( \frac{q_2}{\mu_2} \right) \frac{m_1 m_2}{r_{12}} e^{-r_{12}/\lambda} \]
Searching for New Interactions

\[ V(r) = \alpha G \left( \frac{q_1}{\mu_1} \right) \left( \frac{q_2}{\mu_2} \right) \frac{m_1 m_2}{r_{12}} \ e^{-r_{12}/\lambda} \]

- **Strength relative to gravity**
- **Source**
- **Test mass**
- **Interaction range**

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<tr>
<td>Baryon number ( B )</td>
<td>0.99868</td>
<td>1.001077</td>
<td>-2.429 \times 10^{-3}</td>
</tr>
<tr>
<td>Lepton number ( L )</td>
<td>0.44385</td>
<td>0.459742</td>
<td>-1.565 \times 10^{-2}</td>
</tr>
</tbody>
</table>

Other possible charges: B-L, 3B+L, ..
In frequency space

Free oscillation

Thermal noise

Signal

Filtered data

Noise level at the signal-frequency:
\[ \approx 1000 \text{ nrad/\sqrt{Hz}} \approx 3 \text{ nrad \sqrt{day}} \]
Source integration

Local topography

Difficult and tedious

Preliminary Earth Reference Model

Graph showing West (m/s²) and North (m/s²) against distance (m).